

10.39

$$\stackrel{(10.206)}{Z_n} = \int f_n g^n d\theta$$

積分布関数  $\int_{-\infty}^{\theta} f_n(\theta') d\theta' = F_n(\theta)$   
 $\rightarrow Z_n = \int_{-\infty}^{\theta} f_n g^n d\theta = F_n(\theta) g^n$

( $m^{(n)}$ を $\theta$ で置く)

$$\nabla_{m^{(n)}} \ln Z_n = \frac{\nabla_{m^{(n)}} Z_n}{Z_n} = \frac{1}{Z_n} \nabla_{m^{(n)}} \int f_n g^n d\theta = \frac{1}{Z_n} \int f_n \nabla_{m^{(n)}} g^n d\theta$$

$$= \frac{1}{Z_n} \int f_n \nabla_{m^{(n)}} N(\theta | m^{(n)}, v^{(n)} I) d\theta$$

$$= \frac{1}{Z_n} \int f_n \nabla_{m^{(n)}} \frac{1}{(2\pi)^{\frac{D}{2}}} \frac{1}{(v^{(n)})^{\frac{D}{2}}} \exp \left\{ -\frac{1}{2v^{(n)}} (\theta - m^{(n)})^2 \right\} d\theta$$

$$= \frac{1}{Z_n} \int f_n \frac{1}{(2\pi)^{\frac{D}{2}}} \frac{1}{(v^{(n)})^{\frac{D}{2}}} \frac{1}{v^{(n)} (\theta - m^{(n)})} \exp \left\{ -\frac{1}{2v^{(n)}} (\theta - m^{(n)})^2 \right\} d\theta \quad \leftarrow$$

$$\nabla_m \exp \left\{ -\frac{1}{2v} (\theta - m)^2 \right\}$$

$$= \begin{pmatrix} \partial m_1 \\ \partial m_2 \end{pmatrix} \exp \left\{ -\frac{1}{2v} (\theta^2 - 2\theta m + m^2) \right\}$$

$$= \begin{pmatrix} -\frac{1}{2v} (-2\theta + 2m_1) \exp \left\{ -\frac{1}{2v} (\theta - m)^2 \right\} \\ -\frac{1}{2v} (-2\theta + 2m_2) \exp \left\{ -\frac{1}{2v} (\theta - m)^2 \right\} \end{pmatrix}$$

$$= -\frac{1}{2v} \begin{pmatrix} \partial_1 - m_1 \\ \partial_2 - m_2 \end{pmatrix} (-1) \exp \left\{ -\frac{1}{2v} (\theta - m)^2 \right\}$$

$$= \frac{1}{v} (\theta - m) \exp \left\{ -\frac{1}{2v} (\theta - m)^2 \right\}$$

$$= \frac{1}{Z_n} \int \frac{1}{v^{(n)}} (\theta - m^{(n)}) f_n \frac{1}{(2\pi)^{\frac{D}{2}}} \frac{1}{(v^{(n)})^{\frac{D}{2}}} \exp \left\{ -\frac{1}{2v^{(n)}} (\theta - m^{(n)})^2 \right\} d\theta$$

$$= \frac{1}{v^{(n)}} \int (\theta - m^{(n)}) \frac{1}{Z_n} f_n g^n d\theta$$

$$= \frac{1}{v^{(n)}} \left\{ E_{\frac{1}{Z_n} f_n g^n} [\theta] - m^{(n)} \right\}$$

∴ 7

$$E_{\frac{1}{Z_n} f_n g^n} [\theta] = v^{(n)} \nabla_{m^{(n)}} \ln Z_n + m^{(n)}$$

を得る。

(10.216)  $F'$

$$\nabla_{m^{(n)}} \ln Z_n = \frac{\nabla_{m^{(n)}} Z_n}{Z_n} = \frac{1}{Z_n} \nabla_{m^{(n)}} \left\{ (1-w) N(x_n | m^{(n)}, (v^{(n)}+1) I) + w N(x_n | 0, \alpha I) \right\}$$

← (10.216)

$$= \frac{1}{Z_n} (1-w) \nabla_{m^{(n)}} \frac{1}{(2\pi)^{\frac{D}{2}}} \frac{1}{(v^{(n)}+1)^{\frac{D}{2}}} \exp \left\{ -\frac{1}{2(v^{(n)}+1)} (x_n - m^{(n)})^2 \right\}$$

$$= \frac{1}{Z_n} (1-w) \frac{(x_n - m^{(n)})}{(v^{(n)}+1)} N(x_n | m^{(n)}, (v^{(n)}+1) I)$$

$$= \frac{1}{Z_n} \frac{(x_n - m^{(n)})}{(v^{(n)}+1)} \left\{ z_n - w N(x_n | 0, \alpha I) \right\} \quad \leftarrow (10.216)$$

$$= \frac{(x_n - m^{(n)})}{(v^{(n)}+1)} \left\{ 1 - \frac{w}{Z_n} N(x_n | 0, \alpha I) \right\}$$

F, 7

$$m^{new} = E_{\frac{1}{Z_n} f_n(\theta)}[\theta] = v^n \nabla_{m^n} \ln Z_n + m^n$$

$$= v^n \frac{(x_n - m^n)}{(v^n + 1)} \left\{ 1 - \frac{w}{Z_n} N(x_n | 0, \alpha I) \right\} + m^n$$

$$= v^n \frac{(x_n - m^n)}{(v^n + 1)} p_n + m^n$$

$$p_n = 1 - \frac{w}{Z_n} N(x_n | 0, \alpha I)$$

を得る。

$(v^{new}, Z_n)$

$$\begin{array}{l} v^n \rightarrow v^n - \frac{w}{Z_n} \\ \downarrow \\ \nabla_{m^n} = \partial_{m^n} \end{array}$$

(10, 206)

積分可能でない場合の計算方法  
Σを直接(2Fの場合)計算する

$\checkmark (10, 209)$  Fは  $f_n \partial_{m^n} v^n = f_n \partial_{m^n} z_n$

$$\nabla_{m^n} \ln Z_n = \frac{\partial_{m^n} Z_n}{Z_n} = \frac{1}{Z_n} \partial_{m^n} \int f_n q^n d\theta = \frac{1}{Z_n} \int f_n \partial_{m^n} q^n d\theta$$

$$= \frac{1}{Z_n} \int f_n \partial_{m^n} N(\theta | m^n, v^n I) d\theta$$

$$\frac{2}{2v^n} \frac{1}{(2\pi)^{\frac{n}{2}}} \frac{1}{(v^n)^{\frac{n}{2}}} \exp\left\{-\frac{1}{2v^n} (\theta - m^n)^2\right\}$$

$$= \left\{ -\frac{D}{2v^n} + \frac{1}{2} \left( \frac{\theta - m^n}{v^n} \right)^2 \right\} \frac{1}{Z_n} \int f_n q^n d\theta$$

$$= \frac{1}{2v^n} \left( -\frac{D}{2} \right) \left( \frac{1}{v^n} \right)^{\frac{n}{2}-1} \exp\left\{-\frac{1}{2v^n} (\theta - m^n)^2\right\}$$

$$= -\frac{D}{2v^n} N(x_n | m^n, v^n I) + \frac{1}{2} \left( \frac{\theta - m^n}{v^n} \right)^2 N(x_n | m^n, v^n I)$$

$$= -\frac{D}{2v^n} + \frac{1}{2(v^n)^p} E_{\frac{1}{Z_n} f_n q^n} [(\theta - m^n)^T (\theta - m^n)]$$

$$= -\frac{D}{2v^n} + \frac{1}{2(v^n)^p} N(x_n | m^n, v^n I)$$

$$= -\frac{D}{2v^n} + \frac{1}{2(v^n)^p} E_{\frac{1}{Z_n} f_n q^n} [\theta^T \theta - 2\theta^T m^n + (m^n)^T m^n]$$

$$= -\frac{D}{2v^n} + \frac{1}{2(v^n)^p} \left\{ E_{\frac{1}{Z_n} f_n q^n} [\theta^T \theta] - 2 E_{\frac{1}{Z_n} f_n q^n} [\theta]^T m^n + (m^n)^T m^n \right\}$$

∴  $E_{\frac{1}{Z_n} f_n q^n} [\theta^T \theta]$

$$2(v^n)^2 \nabla_{m^n} \ln Z_n = -D v^n + E[\theta \theta] - 2 E[\theta]^T m^n + (m^n)^2$$

$$\therefore E_{\frac{1}{Z_n} f_n q^n} [\theta^T \theta] = 2(v^n)^2 \nabla_{m^n} \ln Z_n + D v^n + 2 E_{\frac{1}{Z_n} f_n q^n} [\theta]^T m^n - (m^n)^2$$

を得る。

(10.2(6) 答)

✓ (10.2(6))

$$\begin{aligned}
 \nabla_{\theta} \ln \mathcal{L}_n &= \frac{\partial}{\partial \theta} \ln \mathcal{L}_n = \frac{\partial \mathcal{L}_n}{\partial \theta} = \frac{1}{\mathcal{L}_n} \frac{\partial}{\partial \theta} \left\{ (-w) N(x_n | m^n, (w^n + 1) I) + w N(x_n | 0, \alpha I) \right\} \\
 &= \frac{1}{\mathcal{L}_n} (-w) \frac{\partial}{\partial \theta} N(x_n | m^n, (w^n + 1) I) \\
 &= \frac{1}{\mathcal{L}_n} (-w) \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} N(x_n | m^n, (w^n + 1) I) \quad \leftarrow \quad = \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} N(x_n | m^n, (w^n + 1) I) \\
 &= \frac{1}{\mathcal{L}_n} \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \left( \mathcal{L}_n - w N(x_n | 0, \alpha I) \right) \quad \leftarrow \quad \text{← (10.2(6))} \\
 &= \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \left( 1 - \frac{w}{\mathcal{L}_n} N(x_n | 0, \alpha I) \right) \\
 &= \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \rho_n, \quad \rho_n \text{ は } m^{\text{new}} \text{ を使ったと同一}
 \end{aligned}$$

これよ!

$$\begin{aligned}
 E_{\frac{1}{\mathcal{L}_n} f_{\theta}(\mathbf{x})} [\theta^T \theta] &= 2(w^n)^2 \left\{ \frac{-D}{2(w^n + 1)} + \frac{1}{2} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \rho_n + D w^n + 2 E[\theta]^T m^n - (m^n)^2 \\
 &= \left\{ \frac{-D(w^n)^2}{(w^n + 1)} + (w^n)^2 \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \rho_n + D w^n + 2 E[\theta]^T m^n - (m^n)^2
 \end{aligned}$$

とある。

$f^{\text{new}}$  は 等方ガウス分布と仮定しているので

$f^{\text{new}} = N(\theta | m^{\text{new}}, V^{\text{new}} I)$  とすると

$$E_{f^{\text{new}}} [(\theta - m^{\text{new}})^T (\theta - m^{\text{new}})] = \text{var}(\theta) = \text{Tr}(V^{\text{new}} I) = D V^{\text{new}}$$

$$\begin{aligned}
 D V^{\text{new}} &= E_{\frac{1}{\mathcal{L}_n} f_{\theta}(\mathbf{x})} [(\theta - E[\theta])^T (\theta - E[\theta])] = E[\theta^T \theta] - (E[\theta])^2 \\
 &= \left\{ \frac{-D(w^n)^2}{(w^n + 1)} + (w^n)^2 \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \rho_n + D w^n + 2 E[\theta]^T m^n - (m^n)^2 - (E[\theta])^2
 \end{aligned}$$

上で求めた  $E[\theta]$  を使つて

$$\begin{aligned}
 V^{\text{new}} &= \left\{ \frac{-D(w^n)^2}{(w^n + 1)} + \frac{(w^n)^2}{D} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \right\} \rho_n + w^n + \frac{2}{D} \left( V^n \frac{x_n - m^n}{w^n + 1} \rho_n + m^n \right)^T m^n - \frac{(m^n)^2}{D} - \frac{1}{D} \left( V^n \frac{x_n - m^n}{w^n + 1} \rho_n + m^n \right)^2 \\
 &= \frac{-D(w^n)^2}{(w^n + 1)} \rho_n + \frac{(w^n)^2}{D} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \rho_n + w^n + \frac{2}{D} V^n \frac{(x_n - m^n)^T}{w^n + 1} \rho_n m^n + \frac{2}{D} (m^n)^2 - \frac{(m^n)^2}{D} - \frac{1}{D} \left( V^n \frac{x_n - m^n}{w^n + 1} \rho_n \right)^2 - \frac{2}{D} V^n \frac{(x_n - m^n)^T}{w^n + 1} \rho_n m^n - \frac{1}{D} (m^n)^2 \\
 &= \frac{-D(w^n)^2}{(w^n + 1)} \rho_n + \frac{(w^n)^2}{D} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 \rho_n + w^n - \frac{1}{D} \left( V^n \frac{x_n - m^n}{w^n + 1} \rho_n \right)^2 \\
 &= V^n - \frac{(w^n)^2}{(w^n + 1)} \rho_n + \frac{(w^n)^2}{D} \left( \frac{x_n - m^n}{w^n + 1} \right)^2 (\rho_n - \rho_n^2) \quad \text{とある。}
 \end{aligned}$$

( $\tilde{f}_n$  を求める)

(10, 207) F'

$$\begin{aligned}\tilde{f}_n &= \mathbb{Z}_n \frac{q^{\text{new}}}{q^n} = \mathbb{Z}_n \frac{N(\theta | m^{\text{new}}, v^{\text{new}} I)}{N(\theta | m^n, v^n I)} \\ &= \mathbb{Z}_n \frac{(2\pi)^{\frac{D}{2}} (v^{\text{new}})^{\frac{D}{2}} \exp \left\{ -\frac{1}{2v^{\text{new}}} (\theta - m^{\text{new}})^2 \right\}}{(2\pi)^{\frac{D}{2}} (v^n)^{\frac{D}{2}} \exp \left\{ -\frac{1}{2v^n} (\theta - m^n)^2 \right\}} \\ &= \mathbb{Z}_n \left( \frac{v^{\text{new}}}{v^n} \right)^{-\frac{D}{2}} \exp \left[ -\frac{1}{2} \underbrace{\left\{ \frac{1}{v^{\text{new}}} (\theta - m^{\text{new}})^2 - \frac{1}{v^n} (\theta - m^n)^2 \right\}}_{①} \right]\end{aligned}$$

指標の中の平方完成する

$$\begin{aligned}① &= \left( \frac{1}{v^{\text{new}}} - \frac{1}{v^n} \right) \theta^2 - 2\theta^T \left( \frac{m^{\text{new}}}{v^{\text{new}}} - \frac{m^n}{v^n} \right) + \frac{(m^{\text{new}})^2}{v^{\text{new}}} - \frac{(m^n)^2}{v^n} \\ &= \frac{1}{v_n} (\theta - m_n)^2 - \frac{(m_n)^2}{v_n} + \frac{(m^{\text{new}})^2}{v^{\text{new}}} - \frac{(m^n)^2}{v^n} \\ \frac{1}{v_n} &= \frac{1}{v^{\text{new}}} - \frac{1}{v^n} \\ m_n &= v_n \left( \frac{m^{\text{new}}}{v^{\text{new}}} - \frac{m^n}{v^n} \right) = v_n \left\{ m^{\text{new}} \left( \frac{1}{v_n} + \frac{1}{v^n} \right) - \frac{m^n}{v^n} \right\} \\ &= m^{\text{new}} + \frac{v_n}{v^n} m^{\text{new}} - \frac{v_n}{v^n} m^n \\ &= (1 + \frac{v_n}{v^n}) m^{\text{new}} + m^n - (1 + \frac{v_n}{v^n}) m^n \\ &= m^n + (1 + \frac{v_n}{v^n}) (m^{\text{new}} - m^n)\end{aligned}$$

を得る。このF')

$$\begin{aligned}\tilde{f}_n &= \mathbb{Z}_n \left( \frac{v^{\text{new}}}{v^n} \right)^{-\frac{D}{2}} \exp \left[ -\frac{1}{2} \left\{ \frac{1}{v_n} (\theta - m_n)^2 - \frac{(m_n)^2}{v_n} + \frac{(m^{\text{new}})^2}{v^{\text{new}}} - \frac{(m^n)^2}{v^n} \right\} \right] \\ &= \mathbb{Z}_n \left( \frac{v^{\text{new}}}{v^n} \right)^{-\frac{D}{2}} \exp \left[ -\frac{1}{2} \left\{ -\frac{(m_n)^2}{v_n} + \frac{(m^{\text{new}})^2}{v^{\text{new}}} - \frac{(m^n)^2}{v^n} \right\} \right] (2\pi)^{\frac{D}{2}} (v_n)^{\frac{D}{2}} N(\theta | m_n, v_n I)\end{aligned}$$

でよ。

( $s_n$  を求めよ)

5, 7

$$s_n = \sum_n \left( \frac{v^{new}}{v^n} \right)^{\frac{D}{2}} \exp \left[ -\frac{1}{2} \left\{ -\frac{(m_n)^2}{v_n} + \frac{(m^{new})^2}{v^{new}} - \frac{(m^n)^2}{v^n} \right\} \right] (2\pi)^{\frac{D}{2}} (v_n)^{\frac{D}{2}}$$

$$= \sum_n \left( \frac{v^{new}}{v^n} \right)^{\frac{D}{2}} \frac{1}{\exp \left[ -\frac{1}{2} \left\{ \frac{(m_n)^2}{v_n} - \frac{(m^{new})^2}{v^{new}} + \frac{(m^n)^2}{v^n} \right\} \right]} (2\pi)^{\frac{D}{2}} (v_n)^{\frac{D}{2}}$$

二二七 (10.220) (10.221) 5')

$$\frac{1}{v^{new}} = \frac{1}{v_n} + \frac{1}{v^n} = \frac{v^n + v_n}{v_n v^n}$$

$$m^{new} = \frac{v^n}{v_n + v^n} (m_n - m^n) + m^n$$

5, 7

$$\begin{aligned} \frac{(m^{new})^2}{v^{new}} &= \frac{v^n + v_n}{v_n v^n} \left\{ \frac{v^n}{v_n + v^n} (m_n - m^n) + m^n \right\}^2 \\ &= \frac{v^n + v_n}{v_n v^n} \left[ \left( \frac{v^n}{v_n + v^n} \right)^2 \{ m_n^2 - 2m_n^T m^n + (m^n)^2 \} + \frac{2v^n}{v_n + v^n} \{ m_n^T m^n - (m^n)^2 \} + (m^n)^2 \right] \\ &= \frac{v^n}{v_n(v_n + v^n)} \{ m_n^2 - 2m_n^T m^n + (m^n)^2 \} + \frac{2}{v_n} \{ m_n^T m^n - (m^n)^2 \} + \frac{v^n + v_n}{v_n v^n} (m^n)^2 \\ &= m_n^2 \frac{v^n}{v_n(v_n + v^n)} - 2m_n^T \left\{ \frac{v^n}{v_n(v_n + v^n)} - \frac{1}{v_n} \right\} m^n + \left\{ \frac{v^n}{v_n(v_n + v^n)} - \frac{2}{v_n} + \frac{v^n + v_n}{v_n v^n} \right\} (m^n)^2 \end{aligned}$$

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$$\begin{aligned} \frac{(m_n)^2}{v_n} - \frac{(m^{new})^2}{v^{new}} + \frac{(m^n)^2}{v^n} &= (m_n)^2 \left\{ \frac{1}{v_n} - \frac{v^n}{v_n(v_n + v^n)} \right\} + 2m_n^T \left\{ \frac{v^n}{v_n(v_n + v^n)} - \frac{1}{v_n} \right\} m^n + \left\{ \frac{1}{v_n} - \frac{v^n}{v_n(v_n + v^n)} + \frac{2}{v_n} - \frac{v^n + v_n}{v_n v^n} \right\} (m^n)^2 \\ &= (m_n)^2 \frac{1}{v_n + v^n} - 2m_n^T \frac{1}{v_n + v^n} m^n + \frac{1}{v_n + v^n} (m^n)^2 \\ &= \frac{1}{v_n + v^n} (m_n - m^n)^2 \end{aligned}$$

$$T_f \in \mathbb{C}^n$$

$v^{new} = \frac{v_n v^n}{v^n + v_n}$

$$S_n = Z_n \left( \frac{v^{new}}{v^n} \right)^{-\frac{D}{2}} \frac{1}{\exp \left[ -\frac{1}{2} \left\{ \frac{1}{v_n + v^n} (m_n - m^n)^2 \right\} \right]} (2\pi)^{\frac{D}{2}} (v_n)^{\frac{D}{2}}$$

$$= Z_n \left( \frac{v_n}{v^n + v_n} \right)^{\frac{D}{2}} \frac{1}{(2\pi)^{\frac{D}{2}} (v_n + v^n)^{\frac{D}{2}} N(m_n | m^n, (v_n + v^n) I)} (2\pi)^{\frac{D}{2}} (v_n)^{\frac{D}{2}}$$

$$= \frac{Z_n}{N(m_n | m^n, (v_n + v^n) I)}$$

を得る。

( $p(D)$  を求めよ)

$$(10.208) F'$$

$\xleftarrow{(10.208)} \quad \xrightarrow{(10.213)}$

$$p(D) \simeq \int_{\mathbb{T}} \prod_{n=1}^N \tilde{f}_n d\theta = \int_{\mathbb{T}} \prod_{n=1}^N S_n N(\theta | m_n, v_n I) d\theta$$

$$= \int_{\mathbb{T}} \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \exp \left\{ -\frac{1}{2v_n} (\theta - m_n)^2 \right\} d\theta$$

$$= \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \int \exp \left[ \sum_{n=1}^N \left\{ -\frac{1}{2v_n} (\theta - m_n)^2 \right\} \right] d\theta$$

$$= \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \int \exp \left[ -\frac{1}{2} \left( \theta^2 \sum_{n=1}^N \frac{1}{v_n} - 2\theta \sum_{n=1}^N \frac{m_n}{v_n} + \sum_{n=1}^N \frac{m_n^2}{v_n} \right) \right] d\theta$$

指數の中を平方完成する

$$p(D) \simeq \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \int \exp \left[ -\frac{1}{2} \left\{ \frac{1}{v^{new}} (\theta - m^{new})^2 - \frac{(m^{new})^2}{v^{new}} + \sum_{n=1}^N \frac{m_n^2}{v_n} \right\} \right] d\theta$$

$$\text{但し } \frac{1}{v^{new}} = \sum_{n=1}^N \frac{1}{v_n}$$

$$m^{new} = v^{new} \sum_{n=1}^N \frac{m^n}{v_n}$$

となる。

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$$P(D) \cong \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \exp \left[ -\frac{1}{2} \left\{ -\frac{(m^{new})^2}{v^{new}} + \sum_{n=1}^N \frac{m_n^2}{v_n} \right\} \right] \int \exp \left\{ -\frac{1}{2} \frac{1}{v^{new}} (\theta - m^{new})^2 \right\} d\theta$$

$$= \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \exp \left[ \frac{1}{2} \left\{ \frac{(m^{new})^2}{v^{new}} - \sum_{n=1}^N \frac{m_n^2}{v_n} \right\} \right] (2\pi v^{new})^{\frac{D}{2}} \int N(\theta | m^{new}, v^{new} I) d\theta \approx 1$$

$$= \left\{ \prod_{n=1}^N S_n (2\pi v_n)^{-\frac{D}{2}} \right\} \exp \left( \frac{1}{2} B \right) (2\pi v^{new})^{\frac{D}{2}}$$

$$B = \frac{(m^{new})^2}{v^{new}} - \sum_{n=1}^N \frac{m_n^2}{v_n}$$

走得3。