

## 2.3.4章の最大推定の結果

$$(2.121) \mu_{ML} = \frac{1}{N} \sum_{n=1}^N z_n$$

$$(2.122) \Sigma_{ML} = \frac{1}{N} \sum_{n=1}^N (z_n - \mu_{ML})(z_n - \mu_{ML})^T$$

ガウス分布は

$$(B.26) Gam(\tau | a, b) = \frac{1}{\Gamma(a)} b^a \tau^{a-1} \exp(-b\tau)$$

ガウス分布の平均と分散は

$$(B.27) E[\tau] = \frac{a}{b}$$

$$(B.28) Var[\tau] = \frac{a}{b^2}$$

① ② (10.29), (10.30) ③

$$E_\tau[\tau] = \frac{a_N}{b_N} = \frac{a_0 + \frac{N+1}{2}}{b_0 + \frac{1}{2} E_M \left[ \sum_{n=1}^N (z_n - \mu)^2 + \lambda_0 (\mu - \mu_0)^2 \right]} \quad \begin{array}{l} \text{← (10.29)} \\ \text{← } \tau \text{ の期待値と } \sum_{n=1}^N E[z_n] \text{ が等しいこと} \\ \text{← } E_\tau[\tau] \neq 1/E_\tau[\tau] \text{ である。} \end{array}$$

$$E_\tau \left[ \sum_{n=1}^N (z_n - \mu)^2 + \lambda_0 (\mu - \mu_0)^2 \right] = E_M \left[ \sum_{n=1}^N z_n^2 - 2\mu \sum_{n=1}^N z_n + \mu^2 N + \lambda_0 \mu^2 - 2\lambda_0 \mu_0 \mu + \lambda_0 \mu_0^2 \right]$$

$$= E_M[M^2](N + \lambda_0) - 2E_M[M](\sum z_n + \lambda_0 \mu_0) + \sum_{n=1}^N z_n^2 - \lambda_0 \mu_0^2 \dots ①$$

5.2

$$\begin{aligned} E_\tau[\tau] &= \frac{a_0 + \frac{N+1}{2}}{b_0 + \frac{1}{2} \{ E_M[M^2](N + \lambda_0) - 2E_M[M](\sum z_n + \lambda_0 \mu_0) + \sum_{n=1}^N z_n^2 - \lambda_0 \mu_0^2 \}} \\ &= \frac{\frac{a_0}{N} + \frac{1}{2} + \frac{1}{2N}}{\frac{b_0}{N} + \frac{1}{2} \{ E_M[M^2](1 + \frac{\lambda_0}{N}) - 2E_M[M](\frac{1}{N} \sum z_n + \frac{1}{N} \lambda_0 \mu_0) + \frac{1}{N} \sum_{n=1}^N z_n^2 - \frac{1}{N} \lambda_0 \mu_0^2 \}} \end{aligned}$$

← 2"

$$\theta_M = N(M/M_N, \lambda_N^{-1})$$

$$E_M[M] = \mu_N = \frac{\lambda_0 \mu_0 + N \bar{x}}{\lambda_0 + N} = \frac{\frac{\lambda_0 \mu_0}{N} + \bar{x}}{\frac{\lambda_0}{N} + 1} \rightarrow \bar{x} (N \rightarrow \infty) \dots ②$$

また

$$E_M[M^2] - E_M[M]^2 = E_M[(M - E_M[M])^2] = \lambda_N^{-1}$$

$$E_M[M^2] = E_M[M]^2 + \lambda_N^{-1} = E_M[M]^2 + \frac{1}{(\lambda_0 + N) E_\tau[\tau]} \rightarrow \bar{x}^2 (N \rightarrow \infty) \dots ③$$

5.2

$$\begin{aligned} E_\tau[\tau] &\rightarrow \frac{\frac{1}{2}(\bar{x}^2 - 2\bar{x} \frac{1}{N} \sum z_n + \frac{1}{N} \sum z_n^2)}{N} \quad (N \rightarrow \infty) \\ &= \frac{N}{N \bar{x}^2 - 2\bar{x} \sum z_n + \sum z_n^2} \\ &= \frac{N}{\sum(\bar{x}^2 - 2\bar{x} z_n + z_n^2)} \\ &= \frac{N}{\sum(z_n - \bar{x})^2} \end{aligned}$$

$$\lim_{n \rightarrow \infty} (f(q_m)) = f(\lim_{n \rightarrow \infty} q_m) \text{ は } \text{OK} \text{ か? } \text{ なぜ?}$$

5.2

$$\left( \lim_{N \rightarrow \infty} E_\tau[\tau] \right)^{-1} = \frac{1}{N} \sum (z_n - \bar{x})^2$$

を得る。したがって (2.122) も成り立つ。

$\triangleright \text{var}(\tau) (= 7.11.2)$

$$\text{var}_\tau(\tau) = \frac{\sigma^2}{bN^2} = \frac{a_0 + \frac{N+1}{2}}{\left( b_0 + \frac{1}{2} E_\mu \left[ \sum_{n=1}^N (x_n - \mu)^2 + \lambda_0 (\mu - \mu_0)^2 \right] \right)^2} \quad \leftarrow (10.29)$$

$$= \frac{\frac{a_0}{N^2} + \frac{1}{2N} + \frac{1}{2N^2}}{\left[ \frac{b_0}{N} + \frac{1}{2} \left\{ E_\mu[\mu^2] \left( 1 + \frac{1}{N} \right) - 2E_\mu[\mu] \left( \frac{1}{N} \sum x_n + \frac{\lambda_0 \mu_0}{N} \right) + \frac{1}{N} \sum x_n^2 - \frac{\lambda_0 \mu_0^2}{N} \right\} \right]^2} \quad \leftarrow ①$$

$$\rightarrow \frac{0}{(E_\mu[\mu^2] - 2E_\mu[\mu] \frac{1}{N} \sum x_n + \frac{1}{N} \sum x_n^2)^2} \quad (N \rightarrow \infty)$$

$$\begin{aligned} &= \frac{0}{(\bar{x}^2 - 2\bar{x}^2 + N\bar{x}^2)^2} \quad \leftarrow ②, ③ \\ &= \frac{0}{\{(1+N)\bar{x}^2\}^2} \\ &= 0 \end{aligned}$$

$\therefore N \rightarrow \infty$  のとき  $\text{var}(\tau)$  の分散は 0 に収束。分散の並数の精度は無限大となる。

↑  
 (3.28) 分散を予測する方法  
 (分散の期待値で行なう)