11.10

サンプルの個数はL個とする

$$E[(z^{(\tau)})^2] = \sum_{z^{(0)}} \cdots \sum_{z^{(\tau)}} \cdots \sum_{z^{(L)}} p(z^{(0)}, \cdots z^{(\tau)}, \cdots z^{(L)}) (z^{(\tau)})^2$$

$$= \sum_{z^{(0)}} \cdots \sum_{z^{(\tau)}} \cdots \sum_{z^{(L)}} p(z^{(0)}) \cdots p(z^{(\tau-1)}|z^{(\tau-2)}) p(z^{(\tau)}|z^{(\tau-1)}) \cdots p(z^{(L)}|z^{(L-1)}) (z^{(\tau)})^2$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) \left[\sum_{z^{(\tau)}} p(z^{(\tau)}|z^{(\tau-1)}) (z^{(\tau)})^2 \left[\cdots \left[\underbrace{\sum_{z^{(L)}} p(z^{(L)}|z^{(L-1)})}_{=1} \right] \right] \right] \right]$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) \left[\sum_{z^{(\tau)}} p(z^{(\tau)}|z^{(\tau-1)}) (z^{(\tau)})^2 \right] \right]$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) \left[\underbrace{p(z^{(\tau)}=z^{(\tau-1)}|z^{(\tau-1)})}_{0.5\ (11.34)} (z^{(\tau-1)})^2 + \underbrace{p(z^{(\tau)}=z^{(\tau-1)}+1|z^{(\tau-1)})}_{0.25\ (11.35)} (z^{(\tau-1)}+1)^2 + \underbrace{p(z^{(\tau)}=z^{(\tau-1)}-1|z^{(\tau-1)})}_{0.25\ (11.36)} (z^{(\tau-1)}-1)^2 \right] \right]$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) \left[0.5(z^{(\tau-1)})^2 + 0.25(z^{(\tau-1)}+1)^2 + 0.25(z^{(\tau-1)}-1)^2 \right] \right]$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) \left[(z^{(\tau-1)})^2 + 0.5 \right] \right]$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) (z^{(\tau-1)})^2 \right] + \underbrace{\sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \underbrace{\sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)})}_{=1} \right] 0.5}_{=1}$$

$$= \sum_{z^{(0)}} p(z^{(0)}) \left[\cdots \sum_{z^{(\tau-1)}} p(z^{(\tau-1)}|z^{(\tau-2)}) (z^{(\tau-1)})^2 \right] + 0.5$$

$$= E[(z^{(\tau-1)})^2] + 0.5$$

$\therefore$ $E[(z^{(\tau)})^2] = E[(z^{(\tau-1)})^2] + 0.5$

となる。

これより

$$E[(z^{(\tau)})^2] = E[(z^{(\tau-1)})^2] + 0.5 = \left(E[(z^{(\tau-2)})^2] + 0.5\right) + 0.5$$

$$= \underbrace{\left(E[(z^{(0)})^2]\right.}_{=0} + \underbrace{\left.0.5\right)}_{\tau\text{個}} + \cdots + 0.5 = 0.5\tau$$

となる。

$E[(z^{(0)})^2] = \sum_{z^{(0)}} p(z^{(0)}) (z^{(0)})^2 \underbrace{\sum_{z^{(1)}} p(z^{(1)}|z^{(0)}) \left[\sum_{z^{(2)}} p(z^{(2)}|z^{(1)}) \cdots \right]}_{=1}$

$= \sum_{z^{(0)}} p(z^{(0)}) (z^{(0)})^2$

$= 0$ ← $p(z^{(0)}=0) = 1$