11.16

zとrは独立なので

$$p(r|z) = p(r) = \int p(z,r)\,dz = \int \frac{1}{Z_H} \exp(-H(z,r))\,dz \quad (11.63)$$

$$= \int \frac{1}{Z_H} \exp(-E(z) - K(r))\,dz \quad \leftarrow (11.57)$$

$$= \frac{1}{Z_H} \exp(-K(r)) \int \exp(-E(z))\,dz$$

$$Z_H = \int p(z,r)\,dz\,dr$$
$$= \int \exp(-E(z) - K(r))\,dz\,dr$$
$$= \int \exp(-E(z))\,dz \int \exp(-K(r))\,dr$$

$$= \frac{1}{\int \exp(-E(z))\,dz \int \exp(-K(r))\,dr} \exp(-K(r)) \int \exp(-E(z))\,dz$$

$$= \frac{1}{\int \exp(-K(r))\,dr} \exp(-K(r))$$

$$= \frac{1}{(2\pi)^{\frac{3N}{2}}} \exp\left(-\frac{1}{2}\sum_i r_i^2\right) \quad (11.56)$$

$$\int \exp(-K(r))\,dr = \int \exp\left(-\frac{1}{2}\sum_i r_i^2\right) dr \quad (11.56)$$
$$= (2\pi)^{\frac{3N}{2}} \int \frac{1}{(2\pi)^{\frac{3N}{2}}} \exp\left(-\frac{1}{2} r^T r\right) dr$$
$$= (2\pi)^{\frac{3N}{2}} \int N(r|0, I)\,dr$$
$$= (2\pi)^{\frac{3N}{2}}$$

$$= \frac{1}{(2\pi)^{\frac{3N}{2}}} \exp\left(-\frac{1}{2} r^T r\right)$$

$$= N(r|0, I)$$

となり $p(r|z)$ はガウス分布である。