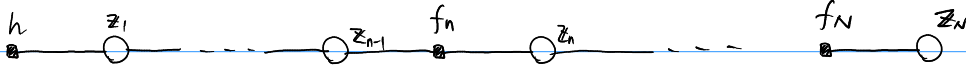


13.18

以下の因子グラフについて



$$f_n(z_{n-1}, z_n) = p(u_n) p(z_n | u_n, z_{n-1}) p(x_n | u_n, z_n)$$

$$h(z_1) = p(u_1) p(z_1 | u_1) p(x_1 | z_1, u_1)$$

右向きメッセージは (8.66), (8.69) より

$$\begin{aligned} \mu_{f_n \rightarrow z_n}(z_n) &= \sum_{z_{n-1}} f_n(z_{n-1}, z_n) \mu_{z_{n-1} \rightarrow f_n}(z_{n-1}) && \text{(8.66)} \\ &= \sum_{z_{n-1}} f_n(z_{n-1}, z_n) \mu_{f_{n-1} \rightarrow z_{n-1}}(z_{n-1}) && \text{(8.69) より} \\ & && \mu_{z_{n-1} \rightarrow f_n}(z_{n-1}) = \mu_{f_{n-1} \rightarrow z_{n-1}}(z_{n-1}) \end{aligned}$$

を得る。ここで

$$\alpha(z_n) \equiv \mu_{f_n \rightarrow z_n}(z_n) \quad (n \geq 2)$$

と定義すると

$$\alpha(z_n) = \sum_{z_{n-1}} f_n(z_{n-1}, z_n) \alpha(z_{n-1}) = \sum_{z_{n-1}} p(u_n) p(z_n | u_n, z_{n-1}) p(x_n | u_n, z_n) \alpha(z_{n-1})$$

を得る。また (8.71) より

$$\mu_{h \rightarrow z_1}(z_1) = h(z_1)$$

を得る

$$\alpha(z_1) \equiv \mu_{h \rightarrow z_1}(z_1)$$

と定義すると

$$\alpha(z_1) = h(z_1) = p(u_1) p(z_1 | u_1) p(x_1 | z_1, u_1)$$

を得る。

左向き の X, y 列- z は、(8.66)、(8.69) より

$$\begin{aligned} \mu_{f_{n+1} \rightarrow z_n}(z_n) &= \sum_{z_{n+1}} f_{n+1}(z_n, z_{n+1}) \mu_{z_{n+1} \rightarrow f_{n+1}}(z_{n+1}) && \text{(8.66)} \\ &= \sum_{z_{n+1}} f_{n+1}(z_n, z_{n+1}) \mu_{f_{n+2} \rightarrow z_{n+1}}(z_{n+1}) && \text{(8.69) より} \end{aligned}$$

$\mu_{z_{n+1} \rightarrow f_{n+1}}(z_{n+1}) = \mu_{f_{n+2} \rightarrow z_{n+1}}(z_{n+1})$

と得る。ここで

$$\beta(z_n) \equiv \mu_{f_{n+1} \rightarrow z_n}(z_n) \quad (n < N)$$

と定義すると

$$\begin{aligned} \beta(z_n) &= \sum_{z_{n+1}} f_{n+1}(z_n, z_{n+1}) \beta(z_{n+1}) \\ &= \sum_{z_{n+1}} p(u_{n+1}) p(z_{n+1} | u_{n+1}, z_n) p(x_{n+1} | u_{n+1}, z_{n+1}) \beta(z_{n+1}) \end{aligned}$$

と得る。また (8.70) より

$$\mu_{z_N \rightarrow f_N}(z_N) = 1$$

ここで

$$\beta(z_N) \equiv \mu_{z_N \rightarrow f_N}(z_N)$$

と定義すると

$$\beta(z_N) = 1$$

と得る。