13.29 解答みた

(13.99)の両辺に$\hat{\alpha}(z_n)$をかける

$$c_{n+1}\hat{\alpha}(z_n)\hat{\beta}(z_n) = \hat{\alpha}(z_n)\int \hat{\beta}(z_{n+1})p(x_{n+1}|z_{n+1})p(z_{n+1}|z_n)dz_{n+1}$$

$$\therefore c_{n+1}\gamma(z_n) = \int \hat{\beta}(z_{n+1})p(x_{n+1}|z_{n+1})\hat{\alpha}(z_n)p(z_{n+1}|z_n)dz_{n+1} \quad \cdots (13.99)'$$

となる。ここで

$$\hat{\alpha}(z_n)p(z_{n+1}|z_n) = \overset{(13.84)}{N(z_n|\mu_n, V_n)}\overset{(13.75)}{N(z_{n+1}|Az_n, \Gamma)}$$

$$= N(z_{n+1}|A\mu_n, \Gamma + AV_nA^T) \times \quad \leftarrow (2.115)$$

$$N(z_n|Y(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), Y) \leftarrow (2.116) \quad ただし\ Y = (V_n^{-1} + A^T\Gamma^{-1}A)^{-1} \leftarrow (2.117)$$

$$= N(z_{n+1}|A\mu_n, \overset{(13.88)}{P_n})N(z_n|Y(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), Y)$$

なので、これより(13.99)'は $\cdots (13.99)''$

$$c_{n+1}\gamma(z_n) = \int \hat{\beta}(z_{n+1})p(x_{n+1}|z_{n+1})N(z_{n+1}|A\mu_n, P_n)N(z_n|Y(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), Y)dz_{n+1}$$

となる。ここで

$$p(x_{n+1}|z_{n+1})N(z_{n+1}|A\mu_n, P_n) = \overset{(13.76)}{N(x_{n+1}|Cz_{n+1}, \Sigma)}N(z_{n+1}|A\mu_n, P_n)$$

$$= N(x_{n+1}|CA\mu_n, \Sigma + CP_n^{-1}C^T) \times \quad \leftarrow (2.115)$$

$$N(z_{n+1}|M(C^T\Sigma^{-1}x_{n+1} + P_n^{-1}A\mu_n), M) \leftarrow (2.116) \quad ただし\ M = (P_n^{-1} + C^T\Sigma^{-1}C)^{-1} \leftarrow (2.117) \quad \cdots ①$$

ここで

$$M = (P_n^{-1} + C^T\Sigma^{-1}C)^{-1} \overset{(C.7)}{=} P_n - P_nC^T(\Sigma + CP_nC^T)^{-1}CP_n$$

$$= P_n - \overset{(13.92)}{K_{n+1}}CP_n = (I - K_{n+1}C)P_n = \overset{(13.90)}{V_{n+1}} \cdots ②$$

また

$$M(C^T\Sigma^{-1}x_{n+1} + P_n^{-1}A\mu_n) = (P_n^{-1} + C^T\Sigma^{-1}C)^{-1}(C^T\Sigma^{-1}x_{n+1} + P_n^{-1}A\mu_n)$$

$$= (P_n^{-1} + C^T\Sigma^{-1}C)^{-1}C^T\Sigma^{-1}x_{n+1} + (P_n^{-1} + C^T\Sigma^{-1}C)^{-1}P_n^{-1}A\mu_n$$

$$= P_nC^T(CP_nC^T + \Sigma)^{-1}x_{n+1} \leftarrow (C.5) \quad + (I - K_{n+1}C)A\mu_n \leftarrow ②$$

$$= \overset{(13.92)}{K_{n+1}}x_{n+1} + (I - K_{n+1}C)A\mu_n$$

$$= A\mu_n + K_{n+1}(x_{n+1} - CA\mu_n)$$

$$= \mu_{n+1} \leftarrow (13.92)$$

よって①は

$$p(x_{n+1}|z_{n+1})N(z_{n+1}|A\mu_n, P_n)$$

$$= N(x_{n+1}|CA\mu_n, \Sigma + CP_n^{-1}C^T)N(z_{n+1}|\mu_{n+1}, V_{n+1})$$

$$= N(x_{n+1}|CA\mu_n, \Sigma + CP_n^{-1}C^T)\hat{\alpha}(z_{n+1}) \leftarrow (13.84)$$

となる。

これを用いて $(13.99)''$ は

$$c_{n+1}\gamma(z_n)$$

$$= \int \hat{\beta}(z_{n+1}) N(x_{n+1} | CA\mu_n, \Sigma + CP_n^{-1}C^T)\, \hat{\alpha}(z_{n+1}) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1}$$

$$\overset{(13.98)}{=} \int \gamma(z_{n+1}) N(x_{n+1} | CA\mu_n, \Sigma + CP_n^{-1}C^T) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1}$$

$$= N(x_{n+1} | CA\mu_n, \Sigma + CP_n^{-1}C^T) \int \gamma(z_{n+1}) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1}$$

$$= \overset{(13.91)}{c_{n+1}} \int \gamma(z_{n+1}) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1} \cdots (13.99)'''$$

となる。これは $\gamma(z_n)$ の再帰式になっている。

$(13.99)'''$ の積分は

$$\int \gamma(z_{n+1}) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1}$$

$$\overset{(13.98)}{=} \int N(z_{n+1} | \hat{\mu}_{n+1}, \hat{V}_{n+1}) N(z_n | \Psi(A^T\Gamma^{-1}z_{n+1} + V_n^{-1}\mu_n), \Psi)\, dz_{n+1}$$

$$\overset{(2.115)}{=} N(z_n | \Psi(A^T\Gamma^{-1}\hat{\mu}_{n+1} + V_n^{-1}\mu_n), \Psi + \Psi A^T\Gamma^{-1}\hat{V}_{n+1}(\Psi A^T\Gamma^{-1})^T)$$

となる。$(13.99)'''$ に入れて

$$N(z_n | \hat{\mu}_n, \hat{V}_n) = N(z_n | \Psi(A^T\Gamma^{-1}\hat{\mu}_{n+1} + V_n^{-1}\mu_n), \Psi + \Psi A^T\Gamma^{-1}\hat{V}_{n+1}(\Psi A^T\Gamma^{-1})^T)$$

となる。これより

$$\hat{V}_n = \Psi + \Psi A^T\Gamma^{-1}\hat{V}_{n+1}(\Psi A^T\Gamma^{-1})^T$$

$$= V_n - J_nP_nJ_n^T + V_nA^TP_n^{-1}\hat{V}_{n+1}P_n^{-1}AV_n$$

$$= V_n - J_nP_nJ_n^T + J_nP_nP_n^{-1}\hat{V}_{n+1}P_n^{-1}P_nJ_n^T \quad \left\{ \begin{array}{l} AV_n = P_nJ_n^T \\ V_nA^T = J_nP_n \end{array} \right.$$

$$= V_n - J_nP_nJ_n^T + J_n\hat{V}_{n+1}J_n^T$$

$$= V_n + J_n(\hat{V}_{n+1} - P_n)J_n^T \cdots (13.101)$$

を得る。また

$$\hat{\mu}_n = \Psi(A^T\Gamma^{-1}\hat{\mu}_{n+1} + V_n^{-1}\mu_n)$$

$$= \Psi A^T\Gamma^{-1}\hat{\mu}_{n+1} + \Psi V_n^{-1}\mu_n$$

$$= V_nA^TP_n^{-1}\hat{\mu}_{n+1} + (V_n - J_nP_nJ_n^T)V_n^{-1}\mu_n$$

$$\overset{(13.102)}{=} J_n\hat{\mu}_{n+1} + (I - J_nP_nJ_n^TV_n^{-1})\mu_n$$

$$= J_n\hat{\mu}_{n+1} + (I - J_nAV_nV_n^{-1})\mu_n \quad AV_n = P_nJ_n^T$$

$$= J_n\hat{\mu}_{n+1} + (I - J_nA)\mu_n$$

$$= \mu_n + J_n(\hat{\mu}_{n+1} - A\mu_n) \cdots (13.100)$$

を得る。

$$\Psi = (V_n^{-1} + A^T\Gamma^{-1}A)^{-1}$$

$$\overset{(C.7)}{=} V_n - V_nA^T(\Gamma + AV_nA^T)^{-1}AV_n$$

$$\overset{(13.88)}{=} V_n - V_nA^TP_n^{-1}AV_n$$

$$\overset{(13.102)}{=} V_n - J_nAV_n$$

$$= V_n - J_nP_nJ_n^T \quad AV_n = P_nJ_n^T$$

$$\Psi A^T\Gamma^{-1} = (V_n^{-1} + A^T\Gamma^{-1}A)^{-1}A^T\Gamma^{-1}$$

$$\overset{(C.5)}{=} V_nA^T(AV_nA^T + \Gamma)^{-1}$$

$$\overset{(13.88)}{=} V_nA^TP_n^{-1}$$