13.5

ラグランジアンは

(13.17)　　$\sum_k \pi_k = 1$ という制約条件

$$L = Q(\theta, \theta^{old}) + \lambda\left(\sum_{k=1}^{K}\pi_k - 1\right) + \overbrace{\mu_1\left(\sum_{l=1}^{K}A_{1l} - 1\right) + \cdots + \mu_K\left(\sum_{l=1}^{K}A_{Kl} - 1\right)}^{\sum_l A_{jl} = 1 \text{ という制約条件}}$$

$$= \sum_{k=1}^{K}\gamma(z_{1k})\ln\pi_k + \sum_{n=2}^{N}\sum_{j=1}^{K}\sum_{k=1}^{K}\xi(z_{n-1,j}, z_{nk})\ln A_{jk} + \sum_{n=1}^{N}\sum_{k=1}^{K}\gamma(z_{nk})\ln p(x_n|\phi)$$

$$+ \lambda\left(\sum_{k=1}^{K}\pi_k - 1\right) + \mu_1\left(\sum_{l=1}^{K}A_{1l} - 1\right) + \cdots + \mu_K\left(\sum_{l=1}^{K}A_{Kl} - 1\right)$$

これより π_k について停留条件は

$$0 = \frac{\partial L}{\partial \pi_k} = \gamma(z_{1k})\frac{1}{\pi_k} + \lambda$$

$$\therefore \pi_k = -\frac{1}{\lambda}\gamma(z_{1k})$$

となる。ここで

$$1 = \sum_{k=1}^{K}\pi_k = \sum_{k=1}^{K} -\frac{1}{\lambda}\gamma(z_{1k})$$

$$\therefore -\frac{1}{\lambda} = \frac{1}{\sum_{k=1}^{K}\gamma(z_{1k})}$$

よって

$$\pi_k = \frac{\gamma(z_{1k})}{\sum_{k=1}^{K}\gamma(z_{1k})}$$

を得る。

次に A_{jk} についての停留条件は

$$0 = \frac{\partial L}{\partial A_{jk}} = \sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk}) \frac{1}{A_{jk}} + \mu_j$$

$$\therefore\ A_{jk} = \sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk}) \frac{1}{-\mu_j}$$

ここで

$$1 = \sum_{k=1}^{K} A_{jk} = \sum_{k=1}^{K} \sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk}) \frac{1}{-\mu_j}$$

$$\therefore\ -\frac{1}{\mu_j} = \frac{1}{\sum_{k=1}^{K} \sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk})}$$

よって

$$A_{jk} = \frac{\sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk})}{\sum_{k=1}^{K} \sum_{n=2}^{N} \xi(z_{n-1\,j}\ z_{nk})}$$

を得る。