14.14

ラグランジアンは

$$L = Q(\theta, \theta^{old}) + \lambda\left(\sum_{k=1}^{K}\pi_k - 1\right)$$

$$= \sum_{n=1}^{N}\sum_{k=1}^{K}\gamma_{nk}\left\{\ln\pi_k + \ln N(t_n|w_k^T\phi_n, \beta^{-1})\right\} + \lambda\left(\sum_{k=1}^{K}\pi_k - 1\right)$$

π_kについて停留条件は

$$0 = \frac{\partial L}{\partial \pi_k} = \sum_{n=1}^{N}\gamma_{nk}\frac{1}{\pi_k} + \lambda$$

$$\therefore \pi_k = \frac{-1}{\lambda}\sum_{n=1}^{N}\gamma_{nk}$$

λについて停留条件

$$0 = \frac{\partial L}{\partial \lambda} = \sum_{k=1}^{K}\pi_k - 1$$

より

$$1 = \sum_{k=1}^{K}\pi_k = \sum_{k=1}^{K}\frac{-1}{\lambda}\sum_{n=1}^{N}\gamma_{nk}$$

$$\therefore \lambda = -\sum_{n=1}^{N}\sum_{k=1}^{K}\gamma_{nk} \overset{(14.37)}{=} -\sum_{n=1}^{N}\sum_{k=1}^{K}p(z_n=k|t,\phi_n,\theta^{old}) = -\sum_{n=1}^{N}\underbrace{\sum_{z_n}p(z_n|t,\phi_n,\theta^{old})}_{1} = -N$$

よって

$$\pi_k = \frac{1}{N}\sum_{n=1}^{N}\gamma_{nk} \quad \cdots (14.38)$$

を得る。