

7.14

(7.76), (7.81) に於いて $\alpha = \alpha^*$, $\beta = \beta^*$ と置き

$$p(t | \alpha, w, \beta^*) = N(t | y(\alpha), \beta^{*-1}) \quad \dots (7.76)$$

$$p(w | t, X, \alpha^*, \beta^*) = N(w | m^*, \Sigma^*) \quad \dots (7.81)$$

と置く。 $t \in \mathcal{T} \subset m^*, \Sigma^*$ は (7.82), (7.83) に於いて $\alpha = \alpha^*$, $\beta = \beta^*$ と $t \in \mathcal{T}$ のとき。

二重積分

$$p(t, w | \alpha, t, X, \alpha^*, \beta^*) = p(t | \alpha, w, \beta^*) p(w | t, X, \alpha^*, \beta^*) \quad (\because \text{乗法定理})$$

$$= N(t | y(\alpha), \beta^{*-1}) N(w | m^*, \Sigma^*)$$

と置く。 w について周辺化すると

$$p(t | \alpha, t, X, \alpha^*, \beta^*) = \int p(t, w | \alpha, t, X, \alpha^*, \beta^*) dw$$

$$= \int N(t | y(\alpha), \beta^{*-1}) N(w | m^*, \Sigma^*) dw$$

$$= \int N(t | w^T \phi(\alpha), \beta^{*-1}) N(w | m^*, \Sigma^*) dw$$

$$= \int N(t | \phi^T(\alpha) w, \beta^{*-1}) N(w | m^*, \Sigma^*) dw$$

$$= N(t | \phi^T(\alpha) m^*, (\beta^*)^{-1} + \phi^T(\alpha) \Sigma^* (\phi^T(\alpha))^T) \quad \leftarrow (2.115) F'$$

$$= N(t | (m^*)^T \phi(\alpha), (\beta^*)^{-1} + \phi^T(\alpha) \Sigma^* \phi(\alpha)) \quad \leftarrow \phi^T m^* = (m^*)^T \phi \quad (2.65 - 2.67)$$

$$= N(t | (m^*)^T \phi(\alpha), \sigma^2(\alpha)) \quad \dots (7.90)$$

$t \in \mathcal{T} \subset \mathcal{L}$

$$\sigma^2(\alpha) = (\beta^*)^{-1} + \phi^T(\alpha) \Sigma^* \phi(\alpha) \quad \dots (7.91)$$

を得る。