



$$p(x_1, x_2, x_3) = p(x_3 | x_1, x_2) p(x_1) p(x_2)$$

$$\begin{aligned} \int p(x_1, x_2, x_3) dx_1 dx_2 dx_3 &= \int p(x_3 | x_1, x_2) p(x_1) p(x_2) dx_1 dx_2 dx_3 \\ &= \iiint p(x_3 | x_1, x_2) dx_3 p(x_1) dx_2 p(x_2) dx_1 \\ &= \iint p(x_1) dx_2 p(x_1) dx_1 \quad \leftarrow \int p(x_3 | x_1, x_2) dx_3 = 1 \text{ ("")} \\ &= 1 \quad \leftarrow \int p(x_2) dx_2 = 1, \int p(x_1) dx_1 = 1 \text{ ("")} \end{aligned}$$

$$p(x) = \prod_{k=1}^K p(x_k | p_{ak})$$

$$\begin{aligned} \int p(x) dx &= \int \prod_{k=1}^K p(x_k | p_{ak}) dx_1 \dots dx_K \\ &= \iint p(x_K | p_{aK}) dx_K \prod_{k=1}^{K-1} p(x_k | p_{ak}) dx_1 \dots dx_{K-1} \\ &= \int \prod_{k=1}^{K-1} p(x_k | p_{ak}) dx_1 \dots dx_{K-1} \quad \leftarrow \int p(x_K | p_{aK}) dx_K = 1 \\ &= \dots = 1 \quad \leftarrow \int p(x_k | p_{ak}) dx_k = 1 \end{aligned}$$

$k=1 \sim K-1$ の $p(x_k | p_{ak}) = x_k$ は分母なし
 $k=K$ の $p(x_K | p_{aK}) = x_K$ は分子あり、 x_K は分母あり
 したがって $\int p(x_K | p_{aK}) dx_K = 1$

同様にして「順番に総乗の次数を下げた」 $k=K \sim k-1$ の帰納法可