9.26

ノートのタイトル 2016/05/06

(9.17) $\mu_k = \frac{1}{N_k}\sum_{n=1}^{N}\gamma(z_{nk})x_n$

(9.18) $N_k = \sum_{n=1}^{N}\gamma(z_{nk})$

逐次EMアルゴリズムにおいてEステップで$\gamma(z_{mk})$を更新したときMステップの更新式を求める

(9.18)より

$$N_k^{old} = \sum_{n=1}^{N}\gamma^{old}(z_{nk})$$

$$N_k^{New} = \sum_{n=1}^{N}\gamma^{New}(z_{nk})$$

$n \neq m$について$\gamma^{old}(z_{nk}) = \gamma^{New}(z_{nk})$なので

$$N_k^{New} = \sum_{n=1}^{N}\gamma^{old}(z_{nk}) - \gamma^{old}(z_{mk}) + \gamma^{New}(z_{mk}) = N_k^{old} - \gamma^{old}(z_{mk}) + \gamma^{New}(z_{mk}) \quad \cdots(9.79)$$

である。

(9.17)より

$$\mu_k^{old} = \frac{1}{N_k^{old}}\sum_{n=1}^{N}\gamma^{old}(z_{nk})x_n$$

$$\mu_k^{New} = \frac{1}{N_k^{New}}\sum_{n=1}^{N}\gamma^{New}(z_{nk})x_n$$

これより

$$\begin{aligned}
\mu_k^{New} &= \frac{1}{N_k^{New}}\left\{\sum_{n=1}^{N}\gamma^{old}(z_{nk})x_n - \gamma^{old}(z_{mk})x_m + \gamma^{New}(z_{mk})x_m\right\} \\
&= \frac{1}{N_k^{New}}\left\{N_k^{old}\mu_k^{old} + \left(\gamma^{New}(z_{mk}) - \gamma^{old}(z_{mk})\right)x_m\right\} \\
&= \frac{N_k^{old}}{N_k^{New}}\mu_k^{old} + \frac{\left(\gamma^{New}(z_{mk}) - \gamma^{old}(z_{mk})\right)}{N_k^{New}}x_m \\
&= \frac{N_k^{New} - \left(\gamma^{New}(z_{mk}) - \gamma^{old}(z_{mk})\right)}{N_k^{New}}\mu_k^{old} + \frac{\left(\gamma^{New}(z_{mk}) - \gamma^{old}(z_{mk})\right)}{N_k^{New}}x_m \quad \leftarrow (9.79)\text{より} \\
&= \mu_k^{old} + \frac{\left(\gamma^{New}(z_{mk}) - \gamma^{old}(z_{mk})\right)}{N_k^{New}}\left(x_m - \mu_k^{old}\right) \quad \cdots(9.78)
\end{aligned}$$

を得る。