演習5.19

ノートのタイトル　2014/05/07

$y_n = \sigma(a_n)$, $a_n = \sum w_j z_{nj}$

$E = -\sum_n \{ t_n \ln y_n + (1-t_n) \ln (1-y_n) \}$ … (5.23) で K=1のとき

$$\nabla E = -\sum_n \left\{ t_n \frac{\nabla y_n}{y_n} + (1-t_n) \frac{-\nabla y_n}{1-y_n} \right\}$$

ここで

$$\nabla\left(\frac{\nabla y_n}{y_n}\right) = \frac{(\nabla\nabla y_n) y_n - \nabla y_n (\nabla y_n)^T}{y_n^2}$$

$$\nabla\left(\frac{\nabla y_n}{1-y_n}\right) = \frac{(\nabla\nabla y_n)(1-y_n) + \nabla y_n (\nabla y_n)^T}{(1-y_n)^2}$$

$$\therefore \nabla\nabla E = -\sum_n t_n \frac{(\nabla\nabla y_n) y_n - \nabla y_n (\nabla y_n)^T}{y_n^2} + (1-t_n)(-1) \frac{(\nabla\nabla y_n)(1-y_n) + \nabla y_n (\nabla y_n)^T}{(1-y_n)^2}$$

$$= -\sum_n \nabla y_n (\nabla y_n)^T \left\{ \frac{-t_n}{y_n^2} + \frac{-(1-t_n)}{(1-y_n)^2} \right\} + \nabla\nabla y_n \left\{ \frac{t_n}{y_n} + \frac{-(1-t_n)}{1-y_n} \right\}$$

$$= \sum_n \nabla y_n (\nabla y_n)^T \frac{t_n (1-y_n)^2 + (1-t_n) y_n^2}{y_n^2 (1-y_n)^2} - \nabla\nabla y_n \frac{t_n (1-y_n) - (1-t_n) y_n}{y_n (1-y_n)}$$

ここで

$$\frac{\partial y_n}{\partial w_1} = \frac{\partial}{\partial w_1} \sigma\left(\sum w_j z_{nj}\right) = \sigma(1-\sigma) z_{n1} = y_n (1-y_n) \frac{\partial a_n}{\partial w_1} \leftarrow z_{n1} = \frac{\partial a_n}{\partial w_1}$$

なので

$$\nabla y_n = \begin{pmatrix} \partial_{w_1} \\ \partial_{w_2} \end{pmatrix} y_n = y_n (1-y_n) \begin{pmatrix} \frac{\partial a_n}{\partial w_1} \\ \frac{\partial a_n}{\partial w_2} \end{pmatrix} = y_n (1-y_n) \nabla a_n$$

$$\therefore \nabla y_n (\nabla y_n)^T = y_n^2 (1-y_n)^2 \nabla a_n (\nabla a_n)^T$$

また

$$\nabla\nabla y_n = \begin{pmatrix} \partial_{w_1} \\ \partial_{w_2} \end{pmatrix} (\nabla y_n)^T = \begin{pmatrix} \partial_{w_1} \\ \partial_{w_2} \end{pmatrix} y_n (1-y_n) (\nabla a_n)^T$$

$$= (1-y_n) \nabla y_n (\nabla a_n)^T + y_n (-1) \nabla y_n (\nabla a_n)^T + y_n (1-y_n) \nabla\nabla a_n$$

$$= (1-y_n)^2 y_n \nabla a_n (\nabla a_n)^T - (1-y_n) y_n^2 \nabla a_n (\nabla a_n)^T + y_n (1-y_n) \nabla\nabla a_n$$

$\nabla\nabla a_n = 0$ として

$$\nabla\nabla E = \sum_n \nabla a_n (\nabla a_n)^T \{ t_n (1-y_n)^2 + (1-t_n) y_n^2 \} - \nabla a_n (\nabla a_n)^T \{ (1-y_n) - y_n \} \{ t_n (1-y_n) - (1-t_n) y_n \}$$

$=$

$t_n - \cancel{t_n y_n} - y_n + \cancel{t_n y_n}$

$$(1-2y_n)(t_n-y_n)$$

$$= \sum_n \nabla a_n (\nabla a_n)^T \{ \cancel{t_n} - \cancel{2t_n y_n} + \cancel{t_n y_n^2} + y_n^2 - \cancel{t_n y_n^2} - \cancel{t_n} + y_n + \cancel{2t_n y_n} - 2y_n^2 \}$$

$$= \sum_n \nabla a_n (\nabla a_n)^T (y_n - y_n^2) = \sum_n \nabla a_n (\nabla a_n)^T y_n (1-y_n)$$

$$= \sum_n y_n (1-y_n) b b^T , \quad b = \nabla a_n$$

を得る。