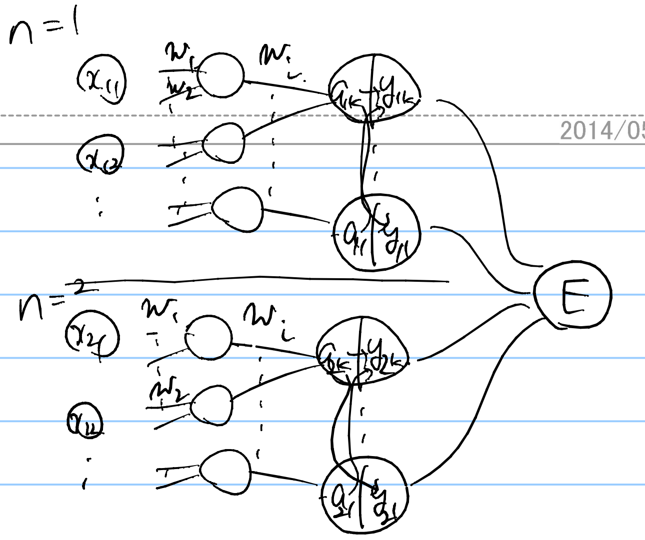
演習5.20 (webの解答見た)



ソフトマックス

$$y_k = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$$

j ↑ 出力ユニットについての和

$$E(w) = -\sum_n^N \sum_k^K t_{nk}\ln y_{nk}, \quad y_{nk}=y_k(x_n,w) \quad \cdots (5.24)$$

$$\frac{\partial E}{\partial w_i} = \sum_n^N\sum_k^K \frac{\partial E}{\partial a_{nk}}\frac{\partial a_{nk}}{\partial w_i}$$

Eはa_{nk}の関数で、a_{nk}はw_iの関数だから
なお、a_{nk}は出力ユニットのa_{nk}を表わしている

$$\frac{\partial E}{\partial a_{nk}} = \sum_m^N\sum_l^K \frac{\partial E}{\partial y_{ml}}\frac{\partial y_{ml}}{\partial a_{nk}} = \sum_l^K \frac{\partial E}{\partial y_{nl}}\frac{\partial y_{nl}}{\partial a_{nk}}$$

ソフトマックス関数の微分

$$\frac{\partial y_l}{\partial a_k} = y_l(I_{lk} - y_k)$$

$$= \frac{\partial E}{\partial y_{nk}} y_{nk}(1-y_{nk}) + \sum_{l\neq k}\frac{\partial E}{\partial y_{nl}}(-y_{nl}y_{nk})$$

$$\frac{\partial E}{\partial y_{nk}} = -t_{nk}\frac{1}{y_{nk}}$$

$$= -t_{nk}\frac{1}{y_{nk}}y_{nk}(1-y_{nk}) + \sum_{l\neq k} -t_{nl}\frac{1}{y_{nl}}(-y_{nl}y_{nk})$$

$$= -t_{nk}(1-y_{nk}) + \sum_{l\neq k} t_{nl}y_{nk}$$

$$= -t_{nk} + y_{nk}\underbrace{\sum_{l=1}^K t_{nl}}_{=1}$$

$$= y_{nk} - t_{nk}$$

よって

$$\frac{\partial E}{\partial w_i} = \sum_n\sum_k (y_{nk}-t_{nk})\frac{\partial a_{nk}}{\partial w_i}$$

$$\frac{\partial y_{nk}}{\partial w_j} = \sum_l \frac{\partial y_{nk}}{\partial a_{nl}}\frac{\partial a_{nl}}{\partial w_j} = \sum_l y_{nk}(I_{kl}-y_{nl})\frac{\partial a_{nl}}{\partial w_j}$$

$$\frac{\partial E}{\partial w_j \partial w_i} = \sum_n\sum_k\left\{\frac{\partial}{\partial w_j}(y_{nk}-t_{nk})\frac{\partial a_{nk}}{\partial w_i} + (y_{nk}-t_{nk})\frac{\partial^2 a_{nk}}{\partial w_j\partial w_i}\right\}$$

$$= \sum_n\sum_k\left\{\sum_l y_{nk}(I_{kl}-y_{nl})\frac{\partial a_{nl}}{\partial w_j}\frac{\partial a_{nk}}{\partial w_i} + (y_{nk}-t_{nk})\frac{\partial^2 a_{nk}}{\partial w_j\partial w_i}\right\}$$

充分にトレーニングされてるネットワークなら $y_{nk}-t_{nk} \fallingdotseq 0$ なので

$$(H)_{ji} = \frac{\partial E}{\partial w_j\partial w_i} \fallingdotseq \sum_n^N\sum_k^K\sum_l^K y_{nk}(I_{kl}-y_{nl})\frac{\partial a_{nl}}{\partial w_j}\frac{\partial a_{nk}}{\partial w_i}$$

を得る。

偏ビ分は変数の他の異なる
偏ビ分同士を
等しいとおかないこと.