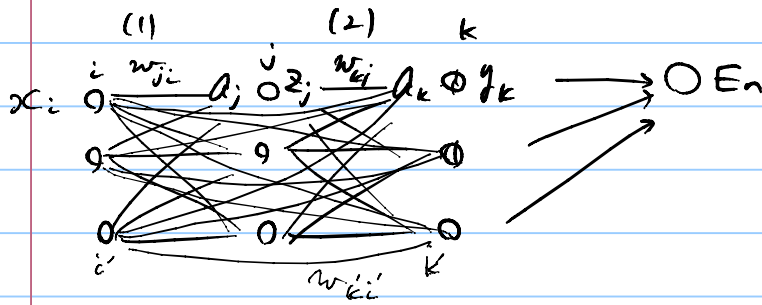


1層ニューラルネットワークの正逆伝播



演習 5.22 と同じ

$$\delta_k = \frac{\partial E_n}{\partial a_k} \quad M_{kk'} = \frac{\partial^2 E_n}{\partial a_k \partial a_{k'}}$$

$$\frac{\partial a_k}{\partial a_j} = w_{kj} h'(a_j), \quad \frac{\partial a_j}{\partial w_{ji}} = x_i, \quad \frac{\partial a_k}{\partial w_{ki}} = z_j$$

$$a_j = \sum_i w_{ji} x_i, \quad z_j = h(a_j)$$

$$a_k = \sum_j w_{kj} z_j + \sum_i w_{ki} x_i, \quad y_k = f(a_k)$$

$$E_n = \frac{1}{2} \sum_k (y_k - t_k)^2, \quad E = \sum_n E_n$$

$$\frac{\partial a_k}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \left\{ \sum_j w_{kj} h(a_j) + \sum_i w_{ki} x_i \right\} = \frac{\partial}{\partial w_{ji}} w_{kj} h(a_j) = w_{kj} x_i h'(a_j)$$

$$\frac{\partial a_k}{\partial w_{ki}} = x_i$$

$$\frac{\partial \delta_k}{\partial w_{ki}} = \frac{\partial}{\partial w_{ki}} \frac{\partial E_n}{\partial a_k}$$

(1) $\frac{\partial E_n}{\partial w_{ki}} = \frac{\partial a_k}{\partial w_{ki}} \frac{\partial E_n}{\partial a_k} = x_i \delta_k$

$$\frac{\partial^2 E_n}{\partial w_{ki} \partial w_{kj}} = \frac{\partial}{\partial w_{ki}} x_i \delta_k = x_i \frac{\partial}{\partial w_{ki}} \delta_k = x_i \frac{\partial a_{k'}}{\partial w_{ki}} \frac{\partial \delta_k}{\partial a_{k'}} = x_i x_{i'} M_{kk'}$$

(2) $\frac{\partial E_n}{\partial w_{kj}} = \frac{\partial a_k}{\partial w_{kj}} \frac{\partial E_n}{\partial a_k} = z_j \delta_k$

$$\frac{\partial^2 E_n}{\partial w_{ki} \partial w_{kj}} = \frac{\partial}{\partial w_{ki}} z_j \delta_k = z_j \frac{\partial}{\partial w_{ki}} \delta_k = z_j x_i M_{kk'}$$

(3) $\frac{\partial E_n}{\partial w_{ji}} = \frac{\partial a_j}{\partial w_{ji}} \frac{\partial E_n}{\partial a_j} = x_i \sum_k \frac{\partial a_k}{\partial a_j} \frac{\partial E_n}{\partial a_k} = x_i \sum_k w_{kj} h'(a_j) \delta_k = x_i h'(a_j) \sum_k w_{kj} \delta_k$

$$\frac{\partial^2 E_n}{\partial w_{ki} \partial w_{ji}} = x_i h'(a_j) \sum_k w_{kj} \frac{\partial \delta_k}{\partial w_{ki}} = x_i h'(a_j) \sum_k w_{kj} \sum_{k'} \frac{\partial a_{k'}}{\partial w_{ki}} \frac{\partial \delta_k}{\partial a_{k'}}$$

$$= x_i h'(a_j) \sum_k w_{kj} \frac{\partial a_k}{\partial w_{ki}} \frac{\partial \delta_k}{\partial a_{k'}} = x_i h'(a_j) \sum_k w_{kj} x_i M_{kk'}$$

$$= x_i x_{i'} \sum_k w_{kj} M_{kk'}$$