5.32

ノートのタイトル 2015/01/04

(目的) $\frac{\partial \tilde{E}}{\partial \eta_j} = \sum_i \{\pi_j - \gamma_j(w_i)\}$ …(5.147)を求める

$$\tilde{E} = E + \lambda \Omega \quad \cdots (5.139)$$

$$\Omega = -\sum_i \ln\left(\sum_j \pi_j N(w_i | \mu_j, \sigma_j^2)\right) \quad \cdots (5.138)$$

より

$$\frac{\partial \Omega}{\partial \pi_j} = -\sum_i \frac{N(w_i | \mu_j, \sigma_j^2)}{\sum_j \pi_j N(w_i | \mu_j, \sigma_j^2)} = -\sum_i \frac{1}{\pi_j} \frac{\pi_j N(w_i | \mu_j \sigma_j^2)}{\sum_j \pi_j N(w_i | \mu_j \sigma_j^2)}$$

ここで

$$\gamma_j(w) = \frac{\pi_j N(w | \mu_j \sigma_j^2)}{\sum_k \pi_k N(w | \mu_k \sigma_k^2)} \quad \cdots (5.140)$$

なので

$$\frac{\partial \Omega}{\partial \pi_j} = -\sum_i \frac{\gamma_j(w_i)}{\pi_j}$$

ここで

$$\pi_j = \frac{\exp(\eta_j)}{\sum_k \exp(\eta_k)} \quad \cdots (5.146)$$

とすると

$$\frac{\pi_k}{\partial \eta_j} = \frac{\delta_{kj} \exp(\eta_j) \sum_k \exp(\eta_k) - \exp(\eta_k) \exp(\eta_j)}{(\sum_k \exp(\eta_k))^2}$$

$$= \delta_{kj} \frac{\exp(\eta_j)}{\sum_k \exp(\eta_k)} - \frac{\exp(\eta_k)}{\sum_k \exp(\eta_k)} \frac{\exp(\eta_j)}{\sum_k \exp(\eta_k)}$$

$$= \delta_{kj} \pi_j - \pi_k \pi_j$$

δ_j について

$$\frac{\partial \Omega}{\partial \eta_j} = \sum_k \frac{\partial \pi_k}{\partial \eta_j} \frac{\partial \Omega}{\partial \pi_k} = \sum_k \left(\delta_{kj} \pi_j - \pi_k \pi_j \right) \left(- \sum_i \frac{\gamma_k(w_i)}{\pi_k} \right)$$

$$= \sum_i \sum_k \pi_k \pi_j \frac{\gamma_k(w_i)}{\pi_k} - \sum_i \sum_k \delta_{kj} \pi_j \frac{\gamma_k(w_i)}{\pi_k}$$

$$= \sum_i \pi_j \underbrace{\sum_k \gamma_k(w_i)}_{=1} - \sum_i \pi_j \frac{\gamma_j(w_i)}{\pi_j}$$

$$= \sum_i \{ \pi_j - \gamma_j(w_i) \} \quad \cdots (5.147)$$

を得る。