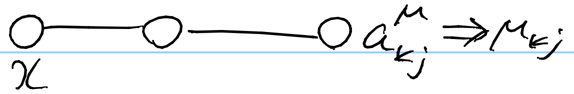
5.35

(問) $\frac{\partial E_n}{\partial a^{\mu}_{kl}} = \gamma_k \left\{ \frac{\mu_{kl} - t_l}{\sigma_k^2} \right\}$ …(5.156) を導く。

ここで μ_k, t はD次元のベクトル。μ_{kl}, t_l はl成分を表す。

(5.153)より

$$E_n = -\ln \sum_{k=1}^{K} \pi_k N(t_n | \mu_k, \sigma_k^2)$$



a^{μ}_{kl} は μ_{kl} を作るのに使われる

$\mu_{kj} = a^{\mu}_{kj}$ ‥(5.152)

なので

$$\frac{\partial E_n}{\partial a^{\mu}_{kl}} = \frac{\partial \mu_{kl}}{\partial a^{\mu}_{kl}} \frac{\partial E_n}{\partial \mu_{kl}} = \frac{\partial E_n}{\partial \mu_{kl}} = -\frac{\pi_k \frac{\partial N_k}{\partial \mu_{kl}}}{\sum_k \pi_k N_k}$$

ここで

$$N(t_n | \mu_k, \sigma_k^2) = \frac{1}{(2\pi)^{\frac{D}{2}}} \frac{1}{\sigma_k} \exp\left\{ -\frac{1}{2} (t - \mu_k)^T \frac{1}{\sigma_k^2} (t - \mu_k) \right\}$$

$$= \frac{1}{(2\pi)^{D/2}} \frac{1}{\sigma_k} \exp\left\{ \frac{-1}{2\sigma_k^2} \left((t_1 - \mu_{k1})^2 + (t_2 - \mu_{k2})^2 + \cdots \right) \right\}$$

なので

$$\frac{\partial N_k}{\partial \mu_{kl}} = N_k \cdot \frac{-1}{2\sigma_k^2} 2 (t_l - \mu_{kl})(-1) = N_k \frac{1}{\sigma_k^2} (t_l - \mu_{kl})$$

これを①に入れて

$$\frac{\partial E_n}{\partial a^{\mu}_{kl}} = -\frac{\pi_k}{\sum_k \pi_k N_k} \times N_k \frac{1}{\sigma_k^2} (t_l - \mu_{kl})$$

$$= \frac{\pi_k N_k}{\sum_k \pi_k N_k} \frac{1}{\sigma_k^2} (\mu_{kl} - t_l) \longleftarrow \gamma_k = \frac{\pi_k N_k}{\sum_k \pi_k N_k} \cdots (5.154)$$

$$= \gamma_k \frac{1}{\sigma_k^2} (\mu_{kl} - t_l) \cdots (5.156)$$

を得る。