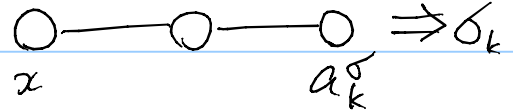


$$(10) \frac{\partial E_n}{\partial a_k^\sigma} = \gamma_{nk} \left(L - \frac{|t_n - \mu_k|^2}{\sigma_k^2} \right) \dots (5.157) \quad \text{E導出}$$

$$\sigma_k = \exp(a_k^\sigma) \dots (5.151)$$



(5.153) より

$$E_n = -\ln \sum_{k=1}^K \pi_k(\lambda_n, w) N(t_n | \mu_k(\lambda_n, w), \sigma_k^2(\lambda_n, w))$$

より

$$\frac{\partial E_n}{\partial a_k^\sigma} = \frac{\partial \sigma_k}{\partial a_k^\sigma} \frac{\partial E_n}{\partial \sigma_k} \dots (1) \quad \text{web9377 見直し修正。} \frac{1}{\sigma_k} \text{ とした。}$$

ここで

$$N(t_n | \mu_k, \sigma_k^2 I) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\sigma_k^2 I|^{1/2}} \exp\left\{-\frac{1}{2} (t_n - \mu_k)^T \frac{1}{\sigma_k^2} I (t_n - \mu_k)\right\}$$

$$= \frac{1}{(2\pi)^{D/2}} \frac{1}{\sigma_k^D} \exp\left\{-\frac{1}{2\sigma_k^2} |t_n - \mu_k|^2\right\}$$

$$|\sigma_k^2 I|^{1/2} = \underbrace{|\sigma_k^2 \times \sigma_k^2 \times \dots|^{1/2}}_{\substack{D \text{ 回} \\ \text{行列式が定数}}}} = |\sigma_k^{2D}|^{1/2} = \sigma_k^D$$

$$\begin{aligned} \frac{\partial N_{nk}}{\partial \sigma_k} &= \frac{1}{(2\pi)^{D/2}} \frac{-D}{\sigma_k^{D+1}} \exp\{\dots\} + \frac{1}{(2\pi)^{D/2}} \frac{1}{\sigma_k^D} \exp\{\dots\} \left\{ -\frac{1}{2} \frac{-2}{\sigma_k^3} |t_n - \mu_k|^2 \right\} \\ &= -\frac{D}{\sigma_k} N_{nk} + N_{nk} \frac{|t_n - \mu_k|^2}{\sigma_k^3} = \frac{N_{nk}}{\sigma_k} \left(-D + \frac{|t_n - \mu_k|^2}{\sigma_k^2} \right) \end{aligned}$$

より (5.157) より

$$\frac{\partial E_n}{\partial \sigma_k} = \frac{\pi_k \frac{\partial N_{nk}}{\partial \sigma_k}}{\sum \pi_k N_{nk}} = \frac{\pi_k \frac{N_{nk}}{\sigma_k} \left(-D + \frac{|t_n - \mu_k|^2}{\sigma_k^2} \right)}{\sum \pi_k N_{nk}}$$

より

$$\frac{\partial \sigma_k}{\partial a_k^\sigma} = \exp(a_k^\sigma) = \sigma_k$$

①に代入

$$\frac{\partial E_n}{\partial a_k^0} = \delta_k \left\{ - \frac{\pi_k \frac{N_{nk}}{\delta_k} \left(-D + \frac{|t_n - M_k|^2}{\delta_k^2} \right)}{\sum \pi_k N_{nk}} \right\}$$

$$= \frac{\pi_k N_{nk}}{\sum \pi_k N_{nk}} \left(D - \frac{|t_n - M_k|^2}{\delta_k^2} \right)$$

$$= \gamma_{nk} \left(D - \frac{|t_n - M_k|^2}{\delta_k^2} \right) \quad \dots (5.157)$$

$$\leftarrow \gamma_{nk} = \frac{\pi_k N_{nk}}{\sum \pi_k N_{nk}} \quad \dots (5.154)$$

を得る