演習5.7

(問題) (5.24)の微分→(5.18), 但し活性化関数はソフトマックス関数

(5.24) $E(w)=-\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\ln y_k(x_n,w) = -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{nk}\ln y_{nk}$

(5.25) $y_k(x,w)=\dfrac{\exp(a_k(x,w))}{\sum_j \exp(a_j(x,w))}$

$$y_{nk}=y_k(x_n,w)=\frac{\exp(a_k(x_n,w))}{\sum_j \exp(a_j(x_n,w))}=\frac{\exp(a_{nk})}{\sum_j \exp(a_{nj})}$$

$$\frac{\partial E}{\partial a_{nk}}=\sum_{i=1}^{N}\sum_{j=1}^{K}\frac{\partial y_{ij}}{\partial a_{nk}}\frac{\partial E}{\partial y_{ij}}$$

ここで

$$\frac{\partial E}{\partial y_{ij}}=t_{ij}\frac{1}{y_{ij}}$$

$$\frac{\partial y_{ij}}{\partial a_{nk}}=\begin{cases}0 & (i\neq n)\\ -y_{nj}y_{nk} & (i=n, j\neq k)\\ y_{nk}(1-y_{nk}) & (i=n, j=k)\end{cases}$$

$$\frac{\partial y_{nj}}{\partial a_{nk}}=\frac{-\exp(a_{nj})\exp(a_{nk})}{\{\sum\exp(a_{nj})\}^2}=-y_{nj}\cdot y_{nk}$$

$$\frac{\partial y_{nk}}{\partial a_{nk}}=\frac{\exp(a_{nk})\sum\exp(a_{nj})-\{\exp(a_{nk})\}^2}{\{\sum\exp(a_{nj})\}^2}$$

$$=y_{nk}-y_{nk}^2=y_{nk}(1-y_{nk})$$

これより

$$\frac{\partial E}{\partial a_{nk}}=\sum_{j=1}^{K}\frac{\partial y_{nj}}{\partial a_{nk}}\frac{\partial E}{\partial y_{nj}}$$

$$=y_{nk}(1-y_{nk})t_{nk}\frac{1}{y_{nk}}+\sum_{j\neq k}-y_{nj}y_{nk}t_{nj}\frac{1}{y_{nj}}$$

$$=(1-y_{nk})t_{nk}-y_{nk}\sum_{j\neq k}t_{nj}$$

$$=t_{nk}-y_{nk}\left(t_{nk}+\sum_{j\neq k}t_{nj}\right)=t_{nk}-y_{nk}$$

($\sum_{j=1}^{K}t_{nj}=1$)

となり(5.18)を得る

連鎖律

$f(w)=f(x_1,w)+f(x_2,w)$

$=f_1(w)+f_2(w)$

$\dfrac{\partial E}{\partial w}=\dfrac{\partial f_1}{\partial w}\dfrac{\partial E}{\partial f_1}+\dfrac{\partial f_2}{\partial w}\dfrac{\partial E}{\partial f_2}$

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